Adiabatic spin pumping through a quantum dot with a single orbital level

Tomosuke Aono*
Department of Physics, Toho University
2-2-1 Miyama, Funabashi, Chiba 274-8510 JAPAN,
and Department of Physics, Ben-Guiron University
Beer-Sheva 84105, ISRAEL
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We investigate an adiabatic spin pumping through a quantum dot with a single orbital energy level under the Zeeman effect. Electron pumping is produced by two periodic time dependent parameters, a magnetic field and a difference of the dot-lead coupling between the left and right barriers of the dot. The maximum charge transfer per cycle is found to be e, the unit charge in the absence of a localized moment in the dot. Pumped charge and spin are different, and spin pumping is possible without charge pumping in a certain situation. They are tunable by changing the minimum and maximum value of the magnetic field.

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Adiabatic electron pumping is the mechanism which produces a finite charge transfer through a system when the system is altered slowly by external parameters and it is returned to its initial state after a certain period [1]. In quantum dot systems, electron pumping has been realized in electron turnstile [2, 3, 4], through which a quantized charge is transferred per cycle under controlled gate voltages by the Coulomb blockade effect. In a recent experiment [5], an adiabatic quantum electron pumping is realized in an open quantum dot under two oscillating gate voltages and zero-source drain voltage, where electrons are transferred by electron interference effect through the system. Many theoretical works have been published in relation to this pumping [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22].

Under magnetic fields, adiabatic spin pumping will occur, which is proposed in the Tomonaga-Luttinger liquid [15] and an open quantum dot [21]. Spin pumping will be very useful in developing of spin dependent transport, especially spin injection methods. Spin injection into semiconductor materials is realized using ferromagnetic metals [23, 24] or magnetic semiconductor [25, 26] contacts. These injections are driven by chemical potential differences across the samples. In contrast, spin pumping works under zero source-drain voltage without magnetic materials. We propose an adiabatic spin pumping through a quantum dot with a single orbital energy level by using an oscillating magnetic field. First we investigate the magnitude of electron pumping and then the separation of pumped charge and spin, including spin pumping without charge pumping as an extreme case.

We consider a quantum dot system as shown in Fig. 1. The dot has a single dot level E_0 and couples to the two leads $\alpha = L, R$ with a tunneling matrix element T_α. The tunneling coupling results in a level broadening of the dot level $\Gamma = \Gamma_L + \Gamma_R$, where $\Gamma_\alpha = \pi\rho|T_\alpha|^2$ with the density of states ρ at the Fermi level in the leads. When a magnetic field B is applied, the dot level splits into $E_\sigma = E_0 - \sigma E_Z$ with the Zeeman energy, $E_Z = g/2\ \mu_B B$ and the spin index σ ($\sigma = \pm$). We assume the Zeeman effect is negligible in the leads [27]. The Coulomb interaction in the dot is taken into account. To describe this system, we adopt the Anderson model [28, 29]:

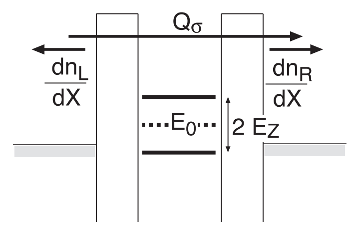

FIG. 1: Schematic view of a quantum dot with a single orbital energy level under a magnetic field.

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\begin{aligned}
H \;=\; & \sum_{k,\sigma,\alpha=L,R} \epsilon_k c^\dagger_{k\sigma\alpha} c_{k\sigma\alpha} + \sum_{\sigma=\pm} E_\sigma d^\dagger_\sigma d_\sigma + U n_+ n_- \\
& + \sum_{k,\sigma,\alpha=L,R} \left(T_\alpha c^\dagger_{k\sigma\alpha} d_\sigma + \text{h.c.} \right). \qquad (1)
\end{aligned}
$$

Here $c^\dagger_{k\sigma\alpha}$ creates an electron with energy ϵ_k and spin σ in lead $\alpha = L, R$, $d^\dagger_\sigma$ creates an electron in the dot with spin σ, $n_\sigma = d^\dagger_\sigma d_\sigma$, and U is the strength of the Coulomb interaction. The Hamiltonian (1) is reduced to the single impurity Anderson model [30] by a unitary transformation for electrons in the leads: $c_{k\sigma} = u^* c_{k\sigma L} + v^* c_{k\sigma R}$, and $\bar{c}_{k\sigma} = -v c_{k\sigma L} + u c_{k\sigma R}$ with $u = T_L/T$, $v = T_R/T$ and $T = \sqrt{|T_L|^2 + |T_R|^2}$ [28]. After the transformation, the modes $\bar{c}_{k\sigma}$ do not couple with electrons in the dot while the modes $c_{k\sigma}$ couple with $d^\dagger_\sigma$ through the tunneling matrix element T.

To this system, we apply adiabatic external sources with a frequency ω. We assume $\omega \ll \Gamma$ while ω is much larger than the Kondo temperature. The former guarantees the adiabatic condition, and the latter means the Kondo effect is suppressed [31]. Accordingly we disregard spin exchange mechanisms and regard spin as a

good quantum number. For simplicity, we discuss the zero temperature limit.

In general, an adiabatic pumping requires two periodic external parameters, X_1 and X_2, with the common frequency ω. Pumped electron charge Q_σ with spin σ from the left to right lead after a period $\tau = 2\pi/\omega$ is given by

$$Q_\sigma = -e \int_0^\tau dt \left(\frac{dn_{L,\sigma}}{dX_1}\frac{dX_1}{dt} + \frac{dn_{L,\sigma}}{dX_2}\frac{dX_2}{dt} \right). \quad (2)$$

Here $dn_{\alpha,\sigma}/dX$ is the emissivity into the lead α ($\alpha = L, R$), which is the number of electrons with spin σ entering into the lead α as a result of the charge redistribution caused by X. It is expressed in terms of the matrix elements of the scattering matrix of the dot, $S_{\sigma;\alpha\beta}$ ($\alpha, \beta = L, R$) [32]:

$$\frac{dn_{\alpha,\sigma}}{dX} = -\frac{1}{2\pi} \sum_{\beta=L,R} \mathrm{Im}\left[S^*_{\sigma;\alpha\beta} \frac{\partial S_{\sigma;\alpha\beta}}{\partial X} \right]. \quad (3)$$

Then Eq. (2) is expressed by a two dimensional integral [9]:

$$Q_\sigma = e \int\int dX_1 dX_2 \Pi_\sigma(X_1, X_2) \quad (4)$$

with

$$\Pi_\sigma(X_1, X_2) = \frac{1}{\pi}\mathrm{Im}\left[\frac{\partial S^*_{\sigma;LL}}{\partial X_1}\frac{\partial S_{\sigma;LL}}{\partial X_2} + \frac{\partial S^*_{\sigma;LR}}{\partial X_1}\frac{\partial S_{\sigma;LR}}{\partial X_2} \right]. \quad (5)$$

A current I_σ with spin σ is given by $I_\sigma = \omega Q_\sigma/2\pi$.

To investigate an adiabatic spin pumping through the dot system, we choose a set of pumping parameters; one is the Zeeman energy, $E_Z(t)$, and the other is the asymmetry factor $p(t)$ defined by $p(t) = (\Gamma_L - \Gamma_R)/\Gamma$, while $\Gamma_L + \Gamma_R$ is kept at a constant value Γ. From its definition, $-1 \le p(t) \le 1$. In general, the matrix elements of the scattering matrix $S_\sigma(X_1, X_2)$ through the dot are given by [29]:

$$\begin{aligned} S_{\sigma;LL/RR}(X_1, X_2) &= 1 - 2i\Gamma_{L/R} G_\sigma, \\ S_{\sigma;LR/RL}(X_1, X_2) &= -2i\sqrt{\Gamma_L \Gamma_R} e^{\pm i\gamma} G_\sigma, \end{aligned} \quad (6)$$

where G_σ is the single-particle Green function for an electron in the dot with spin σ at the Fermi level of the leads, and $\gamma = \mathrm{Arg}(T_L T_R^*)$, which does not appear explicitly in the following discussion. Substituting Eq. (6) into Eq. (5) yields $Q_\sigma = e\int\int dp\, dE_Z\, \Pi_\sigma(E_Z, p)$ with

$$\Pi_\sigma(E_Z, p) = \frac{1}{\pi}\mathrm{Im}\left[\frac{\partial G^*_\sigma}{\partial E_Z} G_\sigma \right]. \quad (7)$$

Since G_σ is the Green function of the single impurity Anderson model [30], it is independent of p. This results in $\Pi_\sigma(E_Z, p) = \Pi_\sigma(E_Z)$. Then the integration over p in the expression of Q_σ is replaced by a constant value; in the following, it is equals to -2.

First we investigate the magnitude of electron pumping. For this purpose, Q_σ is re-expressed by the integration of p and the Friedel phase δ_σ, which is the phase of the transmission coefficient through the dot. It determines the transmission probability T_σ through the dot for electrons with spin σ:

$$T_\sigma = (1 - p^2)\sin^2\delta_\sigma. \quad (8)$$

Furthermore, it satisfies the Friedel sum rule[33, 34]:

$$\langle n_\sigma \rangle = \delta_\sigma/\pi \quad (9)$$

with the occupation number $\langle n_\sigma\rangle = \langle d^\dagger_\sigma d_\sigma\rangle$. Since $\langle n_\sigma\rangle$ is a function of E_Z, $\delta_\sigma = \delta_\sigma(E_Z)$. Using $\exp(2i\delta_\sigma) = 1 - 2i\Gamma G_\sigma$ [33, 34] and substituting it into Eq. (7), we finally obtain

$$Q_\sigma = \frac{2e}{\pi}\int_{\delta_{\sigma,1}}^{\delta_{\sigma,2}} d\delta_\sigma\ \sin^2\delta_\sigma, \quad (10)$$

where we have integrated over p and introduced the lower (upper) limit of the integration, $\delta_{\sigma,1(2)}$. Equation (10) shows that Q_σ has a maximum value e when $\delta_{2,\sigma} = \pi$ and $\delta_{1,\sigma} = 0$, where δ_σ changes so as to run through the resonance of T_σ. [See Eq. (8).] This result coincides with the condition of the maximum pumping [20, 22], which states that it occurs when the trajectory of the pumping parameters encircle the peak of the transmission probability. Note that we have shown this condition in the presence of the Coulomb interactions.

Until now we have assumed the adiabatic condition is satisfied during the pumping cycle. This is, however, not obvious if a localized moment appears in the dot. This is because a magnetization of the localized moment responds sensitively to the change of sign of an applied magnetic field. Thus the time dependence of the magnetization may breaks the adiabatic condition. Accordingly, it will make a difference in electron pumping whether the local moment in the dot appears or not. We discuss this point in the following.

For this purpose, we calculate Q_σ by performing the mean field approximation for the Coulomb interaction term:

$$Un_+n_- = \sum_\sigma U\langle n_{-\sigma}\rangle d^\dagger_\sigma d_\sigma - U\langle n_+\rangle\langle n_-\rangle, \quad (11)$$

where $\langle n_\sigma\rangle = \langle n_\sigma(E_Z)\rangle$ is determined by the self-consistent equations, $\cot(\pi\langle n_\sigma\rangle) = (E_\sigma + U\langle n_{-\sigma}\rangle)/\Gamma$. Using this approximation, we can qualitatively understand how the localized moment appears in the dot, characterized by $M \equiv \langle n_+(0^+)\rangle - \langle n_-(0^+)\rangle$, determined by the value of E_0/Γ for a fixed value of U/Γ [30]. In Fig. 2, M is plotted as a function of E_0/Γ when $U/\Gamma = 10$ with a broken line. When E_0 is large, $M = 0$. At a certain negative value of E_0, M starts to increase.

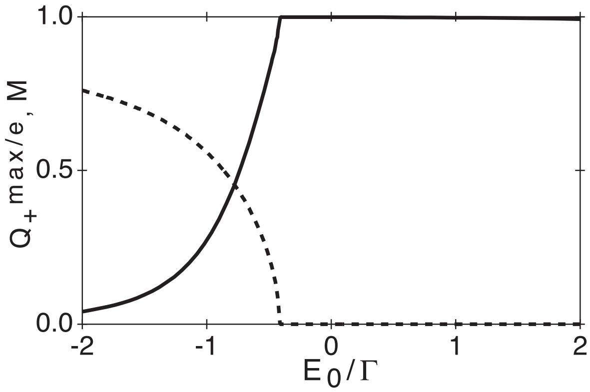

FIG. 2: The maximum value of up-spin pumped charge $Q_+^{\max}$ (solid line) and $M = \langle n_+(0^+)\rangle - \langle n_-(0^+)\rangle$ (broken line) as a function of E_0/Γ when $U/\Gamma = 10$.

After this approximation, Q_σ is given by $Q_\sigma = e\int\int dp\, dE_Z\, \Pi_\sigma(E_Z)$ with

$$\Pi_\sigma(E_Z) = \frac{-1}{\pi}\frac{\Gamma^3\left(\sigma - U\frac{\partial\langle n_{-\sigma}\rangle}{\partial E_Z}\right)}{\left[(E_\sigma + U\langle n_{-\sigma}\rangle)^2 + \Gamma^2\right]^2}, \tag{12}$$

where we have used that $G_\sigma = -1/(E_\sigma + U\langle n_{-\sigma}\rangle - i\Gamma)$. In Fig. 2, the maximum of Q_+, $Q_+^{\max} \equiv e\int_{-\infty}^{\infty} dE_Z \Pi_+(E_Z)$ is plotted as a function of E_0/Γ with a solid line. When $M = 0$, $Q_+^{\max} = e$. As M increases from zero, $Q_+^{\max}$ decreases monotonically.

Now we interpret this result according to the Friedel phase shift argument. If we apply $-E_Z \le E_Z(t) \le E_Z$, the phase shift, $\Delta\delta = \delta_{+,2} - \delta_{+,1}$ is given by

$$\Delta\delta = \int_{\delta_+(-E_Z)}^{\delta_+(0^-)} d\delta_+ + \int_{\delta_+(0^+)}^{\delta_+(E_Z)} d\delta_+ = \pi(M(E_Z) - M) \tag{13}$$

with the magnetization $M(E_Z) = \langle n_+(E_Z)\rangle - \langle n_-(E_Z)\rangle$. The rightmost relation follows from the Friedel sum rule (9) and $\langle n_+(-E_Z)\rangle = \langle n_-(E_Z)\rangle$. Equation (13) means the maximum value of the phase shift is less than π when $M \neq 0$. On the other hand, the maximum value of Q_+ occurs only when $\Delta\delta = \pi$ since the integrand of Eq. (10) is a positive definite function. Hence $Q_+^{\max}$ is suppressed when $M \neq 0$.

Appearance of M, for the dot system under time dependent magnetic fields, is the reflection of the rapid change of $M(E_Z)$ around $E_Z = 0$. In this region, we cannot treat it as an adiabatic variable but rather treat it only as the variable averaged over a certain interval of time because of its rapid change, the value of which is here represented by M. Consequently, the Friedel phase shift in the rapid change region is also replaced by a certain averaged value. In recent theories [16, 17], it has been shown that to see adiabatic pumping clearly, electron coherence throughout the dot is essential and electron pumping is suppressed when the coherence is broken. In the present model, the source of decoherence is the rapid change of the localized moment.

Next, we consider the contributions from both of the up and spins to discuss the separation of pumped charge and spin. First we note an antisymmetric relation between $\Pi_+(E_Z)$ and $\Pi_-(E_Z)$:

$$\Pi_+(E_Z) = -\Pi_-(-E_Z), \tag{14}$$

which is shown as follows: Since $\langle n_+(E_Z)\rangle = \langle n_-(-E_Z)\rangle$,

$$S_+(E_Z, p) = S_-(-E_Z, p), \tag{15}$$

yielding

$$\left.\frac{\partial S_+(X_1,p)}{\partial X_1}\right|_{X_1=E_Z} = -\left.\frac{\partial S_-(X_1,p)}{\partial X_1}\right|_{X_1=-E_Z}. \tag{16}$$

Substituting this relation into Eq. (5) gives Eq. (14). Equation (14) indicates the pumped charge, $Q_{\rm charge} = Q_+ + Q_-$, and the pumped spin, $Q_{\rm spin} = Q_+ - Q_-$ are different in general, determined by the minimum and maximum values of E_Z, $E_Z^{\min}$ and $E_Z^{\max}$.

More importantly, Eq. (14) means that both of up-spin and down-spin pumped charge tend to flow in the opposite directions. This leads the spin pumping without the charge pumping, $Q_{\rm spin} \neq 0$ and $Q_{\rm charge} = 0$, when $E_Z^{\min} = -E_Z^{\max}$. There are a few proposals of spin pumping[15, 21]. In the present system, the spin pumping without charge pumping has the following properties. First, it is always achieved independent of the magnitude of E_Z if $E_Z^{\max} = -E_Z^{\min} \neq 0$. Second, it simply comes from the fact that up and down spins have the opposite signs of the Zeeman energy, not from the presence of electron-electron interactions. These properties will be maintained even for other choices of X_2 instead of p because the asymmetric property similar to Eq.(14) is always valid: $\Pi_+(E_Z, X_2) = -\Pi_-(-E_Z, X_2)$.

Besides the spin pumping without the charge pumping, we can control $Q_{\rm charge}$ and $Q_{\rm spin}$ flexibly by changing the range of E_Z. We illustrate this point by calculating $Q_{\rm charge}$ and $Q_{\rm spin}$ using the mean field approximation (11) for the non-localized moment regime. In Fig. 3(a), $\Pi_+(E_Z)$ and $\Pi_-(E_Z)$ are plotted when $U/\Gamma = 10$ and $E_0/\Gamma = 2$, and in Fig. 3(b), $Q_{\rm charge}$ and $Q_{\rm spin}$ are plotted as a function of $E_Z^{\max}/\Gamma$ with a fixed value of $E_Z^{\min}/\Gamma = -5$. When $E_Z^{\max}$ is small enough, both of $Q_{\rm charge}$ and $Q_{\rm spin}$ are zero. As $E_Z^{\max}$ increases, $Q_{\rm charge}$ and $Q_{\rm spin}$ become finite with opposite signs, where only down spin charge flows from the right to left lead. When $-2 < E_Z^{\max}/\Gamma < 2$, both of $Q_{\rm spin}$ and $Q_{\rm charge}$ take plateau values whose magnitudes are equal to e though the signs are opposite. When $E_Z^{\max}$ increases further, $Q_{\rm charge}$ decreases to zero while $Q_{\rm spin}$ increases to $2e$, where up-spin

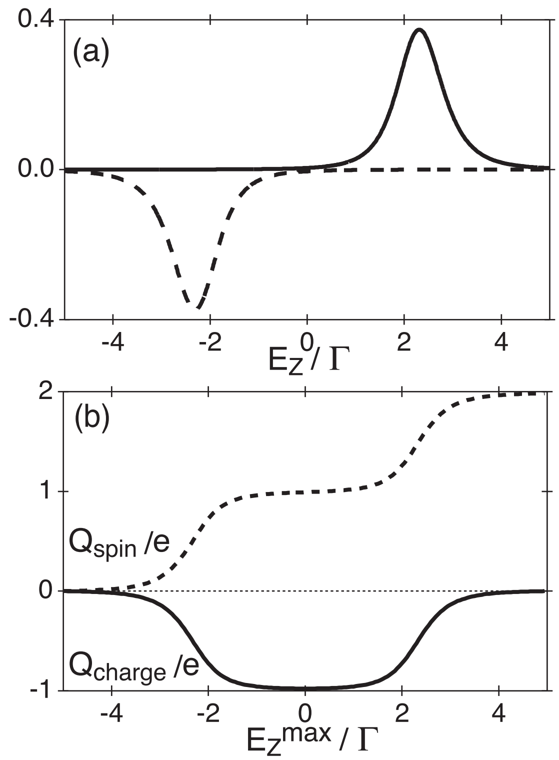

FIG. 3: (a) Plots of Π_σ versus E_Z/Γ: Thick solid (broken) line represents Π_+(Π_-) when $U/\Gamma = 10$ and $E_0/\Gamma = 2.0$. (b)Pumped charge, $Q_{\rm charge} = Q_+ + Q_-$ and pumped spin, $Q_{\rm spin} = Q_+ - Q-$ as a function of $E_Z^{\rm max}/\Gamma$ with a fixed value of $E_Z^{\rm min}/\Gamma = -5$.

charge starts to flow in the opposite direction, from the left to right lead, eventually canceling down-spin charge to achieve the spin pumping without charge pumping. In the same way, we can obtain other possibilities of $Q_{\rm spin}$ and $Q_{\rm charge}$, choosing $E_Z^{\rm min}$ and $E_Z^{\rm max}$ appropriately.

Finally we compare the spin pumping model with another spin current generator using a quantum dot under a finite source-drain voltage and a stationary magnetic field. Since the Zeeman splitting of the dot level acts as a spin filter, we can control the spin flow through the quantum dot by choosing the source-drain voltage appropriately. In this spin filter, however, the up and down spin currents always flow in the same direction. Thus spin current without charge current cannot be achieved. The magnitude of the current of this spin filter is on the order of $e\Gamma$, which is much larger than the one of the spin pumping, which is on the order of $e\omega$.

In conclusion, we investigate an adiabatic spin pumping through a quantum dot with a single orbital level using the Zeeman effect. The maximum value of the unit charge is transferred per cycle. The maximum value is suppressed when the localized moment in the dot appears. Pumped charge and spin are different, and under a certain condition, a spin is pumped with vanishing charge pumping. They are tunable simply by changing the amplitude of magnetic fields. This may introduce flexibility of spin and charge control in semiconductor nano structures.

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* Electronic address: aono@ph.sci.toho-u.ac.jp

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